video (6)

25/4/2020 (Sat) (15)

Note:

Fermat's Theorem

Fermat was not a mathematician. He gave some important results in the field of mathematics.

(1) Fermat's factorization Method :-

Let n be an odd positive integer which is the difference of squares of two integers i.e

$$n = x^2 - y^2$$
$$= (x+y)(x-y)$$

again if $n = ab$, $a \geq b \geq 1$ we can write

$$n = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$$
$$= \frac{a^2 + b^2 + 2ab - a^2 - b^2 + 2ab}{4}$$
$$= \frac{4ab}{4} = ab \quad (\because n = ab)$$

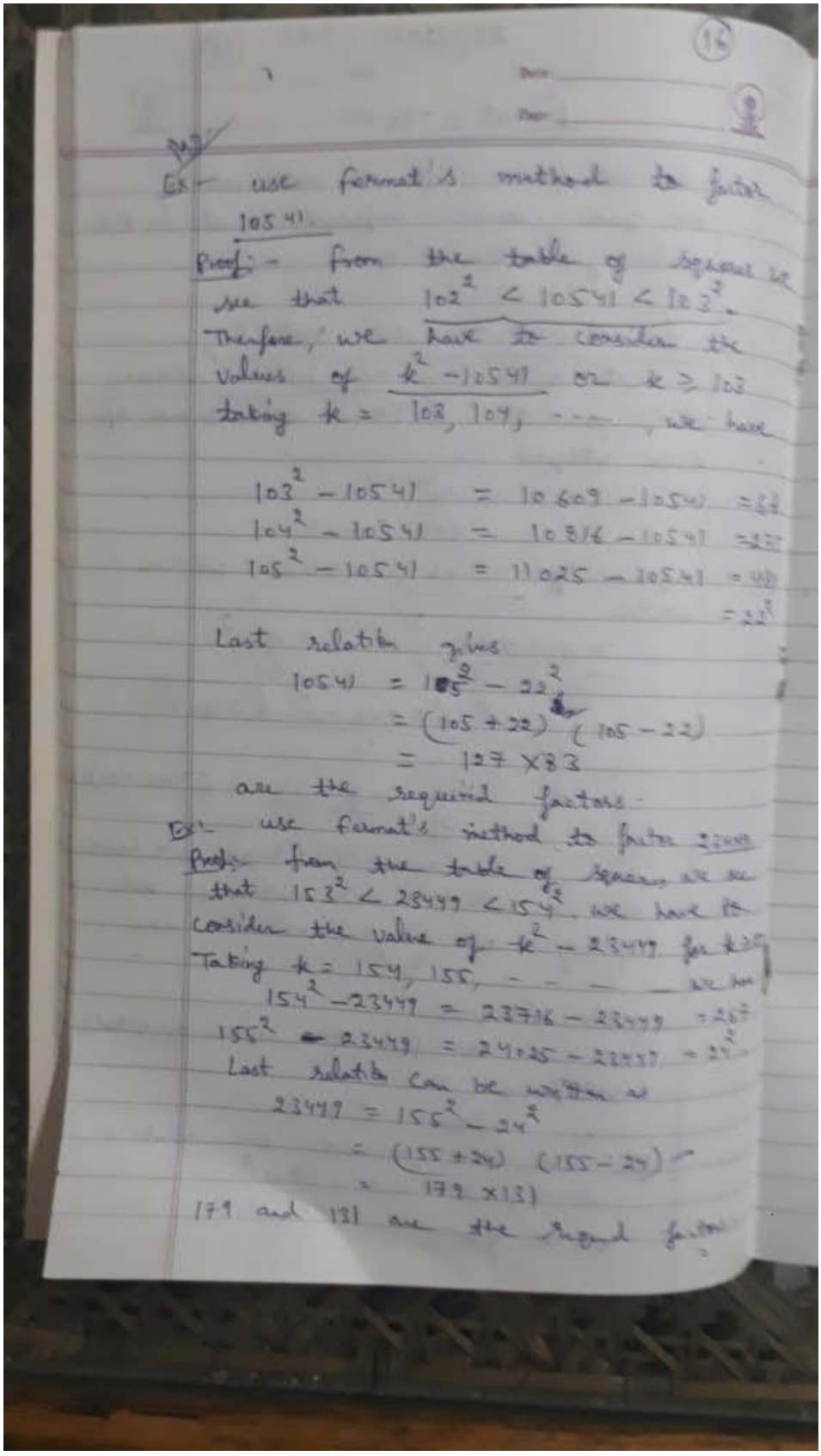
Since n is an odd integer, we have a and b will be odd integers and $\left(\frac{a+b}{2}\right)$ and $\left(\frac{a-b}{2}\right)$ will be non negative integers

we have to find the value of x and y satisfying $n = x^2 - y^2$

or $x^2 - n = y^2$

for this we observe the numbers $x^2 - n$, $(x+1)^2 - n$, $(x+2)^2 - n$, ---

$x = k, k+1$ ---



(15)

Ex:- use fermat's method to factor 10541.

Proof:- from the table of squares we see that $102^2 < 10541 < 123^2$.

Therefore, we have to consider the values of $k^2 - 10541$ for $k \geq 103$ taking $k = 103, 104, \ldots$, we have

$103^2 - 10541 = 10609 - 10541 = 68$

$104^2 - 10541 = 10816 - 10541 = 275$

$105^2 - 10541 = 11025 - 10541 = 484 = 22^2$

Last relation gives

$10541 = 105^2 - 22^2$

$= (105 + 22)(105 - 22)$

$= 127 \times 83$

are the required factors.

Ex:- use fermat's method to factor 23449

Proof:- from the table of squares we see that $153^2 < 23449 < 154^2$. We have to consider the value of $k^2 - 23449$ for $k \geq 154$

Taking $k = 154, 155, \ldots$ we have

$154^2 - 23449 = 23716 - 23449 = 267$

$155^2 - 23449 = 24025 - 23449 = 576 = 24^2$

Last relation can be written as

$23449 = 155^2 - 24^2$

$= (155 + 24)(155 - 24)$

$= 179 \times 131$

179 and 131 are the required factors.